Q 1:Determine wheather each of the following statements is true or false: (10 points)

1) The recursive definition of the set $S = \{1,5,9,13,17,...\}$is $1 \in S; x \in S \to x + 4 \in S$.T

2) $P(n,0) = 0$. **F**

3) $\sum_{k=0}^{n}(-1)^{K}\binom{n}{k} = -1$ ***F***

4) The recurrence relation$a_{n=a_{n-1}+a^2{}_{n-2}}$ is linear. F

5) The recurrence relation$a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3}$ is of degree 3. T

6) The subsets $\{-3,-2,-1,0\}$ and $\{0,1,2,3\}$ are partitions of the set $\{-3,-2,-1,0,1,2,3\}$. **F**

7) **x^2 is $O(x^3)$. T**

8) An undirected graph has an even number of vertices of odd degree.(T)

9-$\sim\left(\exists x(x^2 > 4)\right) = \forall x(x^2 < 4)$.**F**

10- $\{x\} \subset \{x,\{x\}\}$ **. T**

Q 2: Choose the correct answer: (15 points)

1-The minimum number of students required in a class to be sure that at least 10 will receive the same grade if there are 6 possible grades is:

a)60 **b) 10^6** **c) 6^{10}** **d) 55**

2-

$$2 - \sum_{k=0}^{10}\binom{10}{k} =:$$

a)10 b) 1 **c) 1024** d) 0

3- the number of 1-1 functions from a set with 4 elements to a set with 7 elements is:

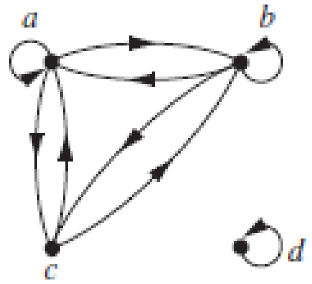
a) 840 b) 11 c)28 d) 3

4- the solution of the recurrence relation($a_n = 3a_{n-1}$) with $a_0 = 2$ is:

a)2.3^n b)2^n c)3^{n+1} d)3^n

5- Relations **R** are defined on the set {1,2,3,4}. Then, one of the following is false :
a) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)} is reflexive, symmetric, antisymmetric and transitive.
b) {(2, 4), (4, 2)} is symmetric only.
c) {(1, 1), (2, 2), (3, 3), (4, 4)} is reflexive, symmetric, antisymmetric and transitive.
d) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)} is not reflexive, not symmetric, not anti-symmetric and not transitive.

6- The relation R represented by the directed graph is shown belo



Then which of the following is true.
a) R is reflexive and symmetric.
b) R is symmetric and transitive.
c) R is symmetric and anti-symmetric.
d) R is anti-symmetric and transitive.

7- The number of relations are there on a set with n elements:
a) $n!$ b) 2^{n^2} c) 2^n d) 2n.

8- The number of edges of K_4 is:

A)10 B)4 C)6 D)8

9- **If** $f(x) = x^2 + x - 3, then\ f^{-1}(9) =$:

a){3} b) {−4} <u>**c){3, −4}**</u> d){−3,4}

10- **If** $|A| = 3, |A \cap B| = 2, |A \cup B| = 10$, **then** $|B| =$:

a)60 b)15 **c)9** d) 5

11- **one of the following pairs are relatively prime:**

a) 6,18 **b)5,17** c)24,18 d) 77,11

12- **if 54 div d=10, 54 mod d=4, then d=:**

a) 9 b)50 **c)5** d)11

13- **let $\boldsymbol{f}: \mathbb{R} \to \mathbb{R}$, then one of the following is not invertible**:

a) f(x)=2x-3 b)f(x)=x+5 c)f(x)=x^3 **d)f(x)=x^2.**

14- **the multiplicative inverse of 3 in $\mathbb{Z}_8$ is:**

a)4 **b)3** c)8 d)5

15) **the number of positive integers not exceeding 100 and divisible by 8 is:**

a) 13 **b) 12** c) 80 d) 92

Q 3: Solve the following questions: (25 points)

1- Use mathematical induction to show that

$$1 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$$

Solution: $P(0)$ is true because $2^0 = 1 = 2^1 - 1$
For the inductive hypothesis, we assume that $P(k)$ is true for an arbitrary non negative integer k
s.t.

$$1 + 2^1 + 2^2 + \cdots + 2^k = 2^{k+1} - 1$$

Taking

$$1 + 2^1 + 2^2 + \cdots + 2^k + 2^{k+1}$$
$$= (2^{k+1} - 1) + 2^{k+1}$$
$$= 2^{k+2} - 1$$

Thus $P(k + 1)$ is also true.

Hence $P(n)$ is true for all non-negative integers n.

2- From a group of 7 men, 3 men are to be selected to form a committee.In how many ways can it be done.

Sol: $\binom{7}{3}$

3- Write the expansion of $(2x - y)^4$.

Sol: Binomial theorem

4- Solve recurrence relation together with the initial condition given $a_n = 2a_{n-1}$ for ≥ 1 , $a_0 = 3$?

$a_n = 3.2^n$.

4− Let m be an integer with m > 1. Show that the congruence modulo m relation

R = {(a, b) | a ≡ b (mod m)} is an equivalence relation on the set of integers.

Ans) Reflexive: a ≡ a (mod m) is true because m |(a-a). i.e. (a, a) ∈ **R.**

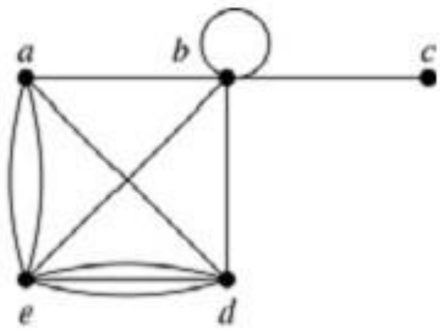
Symmetric: If (a, b) ∈ **R** then a ≡ b (mod m), i.e. m |(a-b). implies m |(b-a)

So b ≡ a (mod m), hence (b,a) ∈ **R.**

Transitive: If (a, b), (b,c) ∈ **R** then a ≡ b (mod m) and b ≡ c (mod m), i.e. m |(a-b) and m |(b-c). implies m |[(a-b)+ (b-c)].implies m |(a-c).

So a ≡ c (mod m), hence (a,c) ∈ **R.**

5- What are the degrees and neighborhoods of the vertices in the graph H?



H

Solution

H:deg(a)=4,deg(b)=deg(e)=6,deg(c)=1,deg(d)=5

N(a)={ b,d,e } N(b)={ a,b,c,de},N(c)={b}

N(d)={a,b,e,} N(e)={ a,b,d}.